Incorporating sediment-transport capabilities to DSM2

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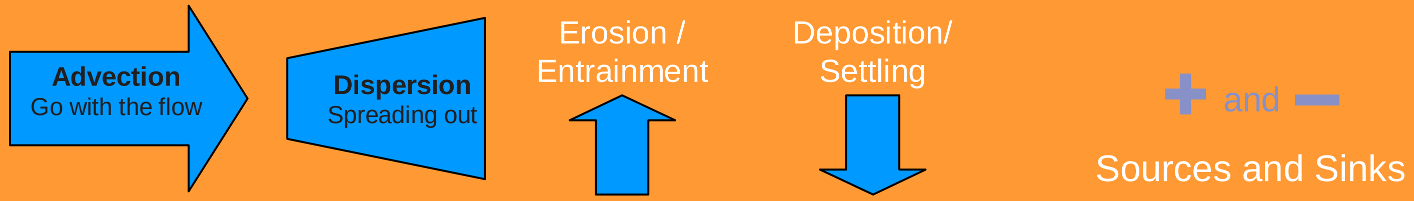
Background



Advection | Dispersion | Erosion | Deposition | Source/Sink | Tributaries

Progress to Date: Single Channel

Advection | Dispersion | Erosion

Next step: Complete single channel model

Advection | Dispersion | Erosion | Deposition | Source/Sink | Tributaries

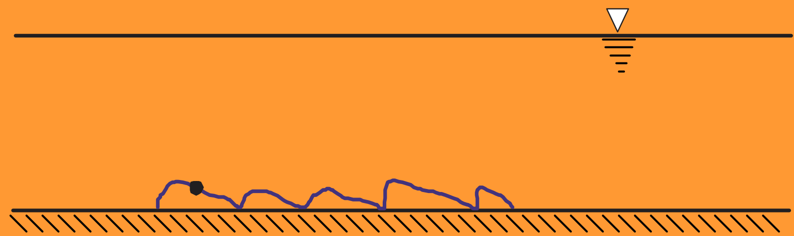
Next step: Extend model to a channel network

Modes of sediment transport

1-Bed Load

Mainly empirical formulas

Lagrangian solution for each particle



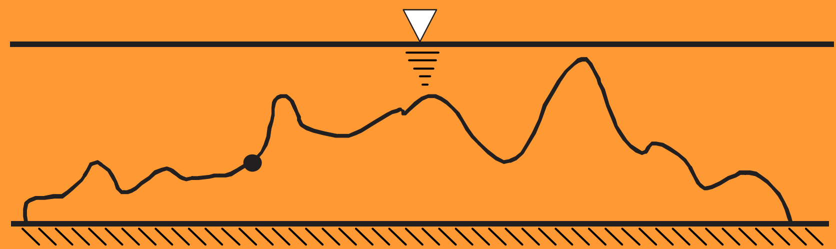
2-Suspended Load

Advection-Dispersion-Sink/Source

3-Wash Load

There is a third mode of sediment transport called wash load, whereby very fine particles are transported downstream with very little interaction with the bed sediments.

During floods, the wash load is deposited in the floodplains (usually ignored in numerical simulations).



Modes of sediment transport

Sediment transport as bed-load in rivers

Source: Prof. Dietrich's website

Modes of sediment transport in the Delta

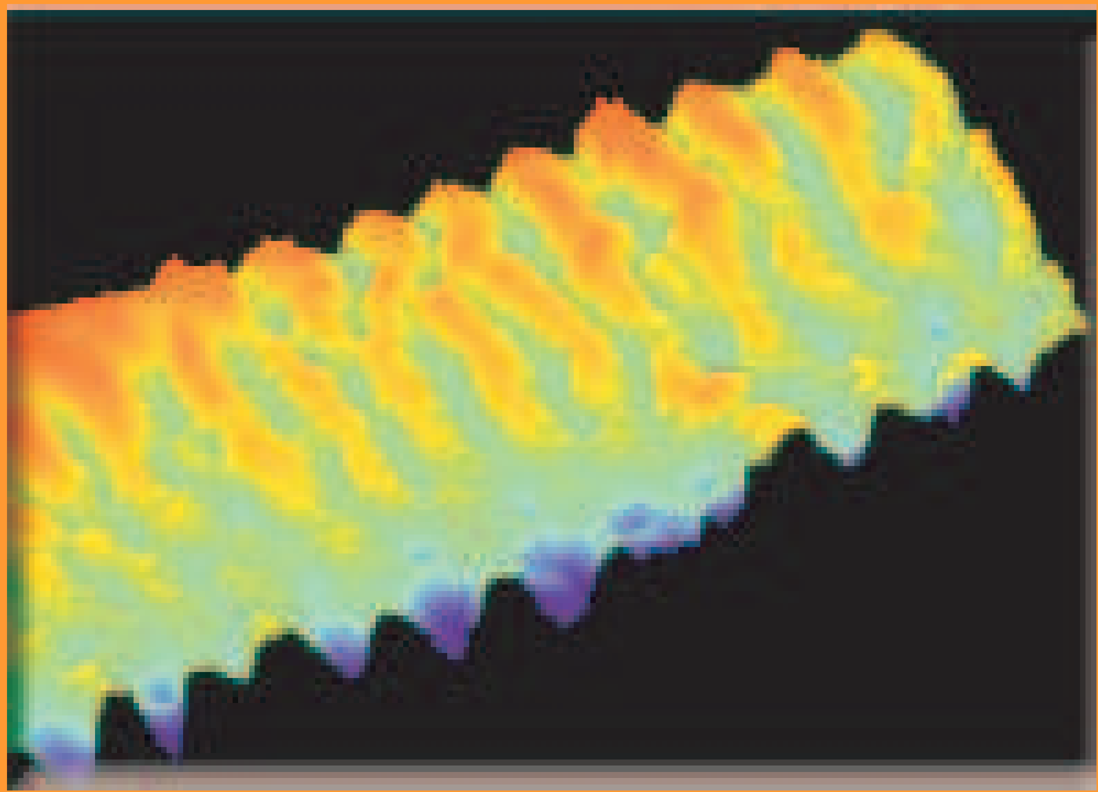
We find the transport of sediment as bed-load and in suspension. Not much information exists about wash load.

In large portions of the Delta, the sediment can be cohesive.

The USGS has numerous stations in which sediment in suspension is monitored via optical backscatter sensors (OBS).

Modes of sediment transport in the Delta

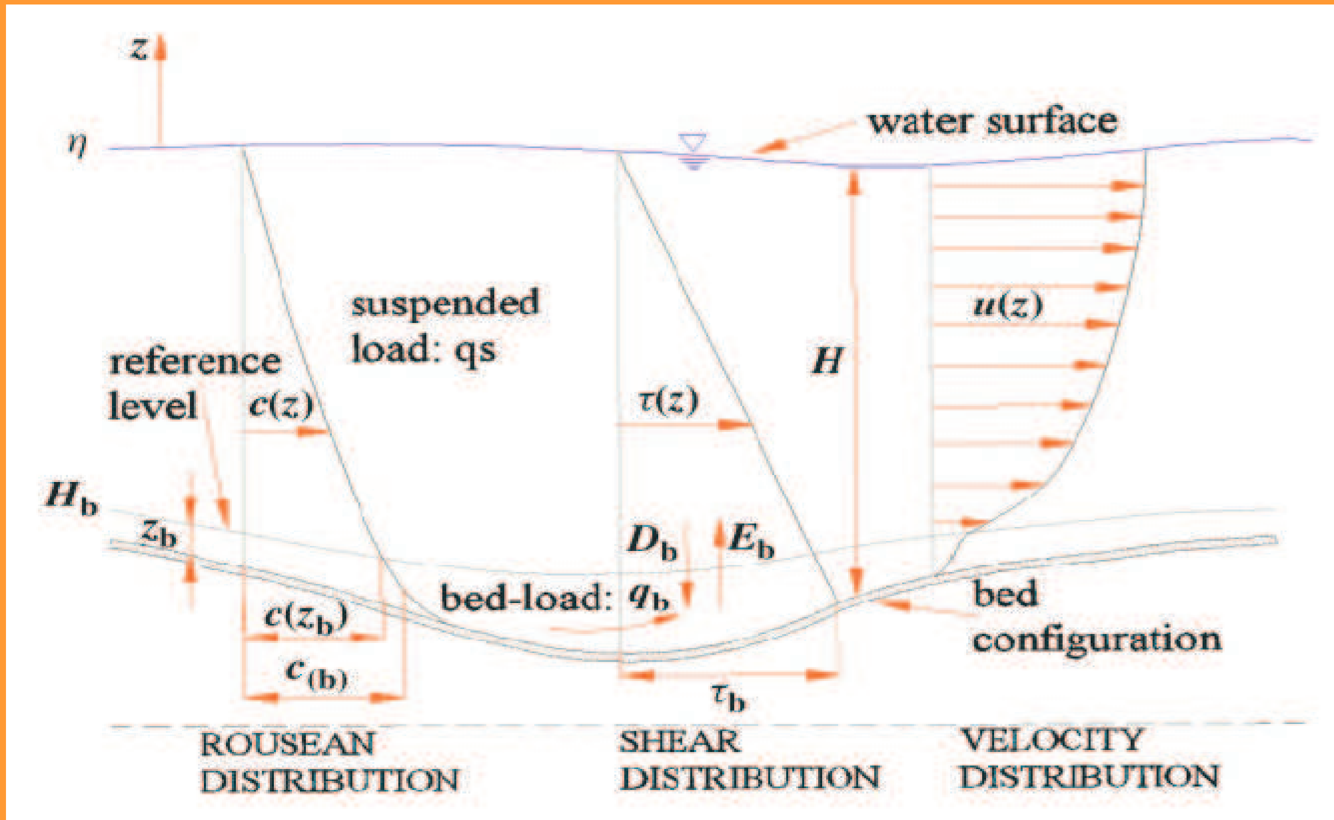
Bed-forms at Garcia Bend in Winter 2000.



Source: USGS website

Mathematical treatment of the problem

Tracking individual particles is not feasible for a system of the size of the Delta. Then, we need to use the continuum approach.



Source: Abad et al., 2007

Mathematical treatment of problem: Sediment in suspension

Sediment transport in suspension:

$$\frac{\partial (A\,C_s)}{\partial t} + \frac{\partial (Q\,C_s)}{\partial s} = \frac{\partial}{\partial s}\left[A\,K_s\,\frac{\partial C_s}{\partial s}\right] + E - D + q_L\,C_L + S/S$$

A : cross-sectional wetted area (m²)

C_s : volumetric cross-sectional-averaged concentration of sediment in suspension (-)

Q : flow discharge (m³/s)

S/S : non-point sources/sinks (m²/s)

K_s : dispersion coefficient (m²/s)

E and D : entrainment rate of sediment into suspension and deposition rate of sediment per unit width, respectively (m²/s)

q_L and C_L : lateral discharge (m²/s), and concentration (-), respectively

Mathematical treatment of the problem: bed-load transport

Sediment transport as bed-load:

$$\frac{q_b}{\sqrt{R\ g\ d_P^3}} = f\left(excess\ shear\ stress\right)$$

q_b : bed-load solid discharge per unit width (m²/s)

$R = \dfrac{\left(\rho_s - \rho\right)}{\rho}$: specific gravity (-)

d_p : sediment particle diameter (m)

g : acceleration of gravity (m/s²)

The equation for sediment in suspension comes from the integration in the cross section up to z_b.

Mathematical treatment of the problem: Entrainment and Deposition

$$E = E_s \; w_s \; B$$

$$D = C_{sl} \; w_s \; B$$

C_{sl} : sediment concentration at the bottom (-)
w_s : settling velocity

Recent developments in sediment transport refer to several active layers, which could be incorporated in a second stage of the model development.

Numerical Method: Operator Splitting

- 2nd order accurate Strang type splitting algorithm

$$c_t = -uc_x + Dc_{xx} + R$$

$$1)\quad c^*_t = -uc^*_x \qquad c^*(t_n, x) = c(t_n, x), \qquad t \in [t_n, t_{n+1/2}]$$

$$2)\quad c^{**}_t = Dc^{**}_x + R \qquad c^{**}(t_n, x) = c^*(t_{n+1/2}, x), \quad t \in [t_n, t_{n+1}]$$

$$3)\quad c^*_t = -uc^*_x \qquad c^*(t_n, x) = c^{**}(t_n, x), \qquad t \in [t_{n+1/2}, t_{n+1}]$$

Numerical Method: Diffusion

- 2nd order, implicit $\frac{\partial(AC_s)}{\partial t} = \frac{\partial}{\partial x}(AK_s \frac{\partial C_s}{\partial x})$

$$\begin{pmatrix} \frac{-\theta\Delta t}{\Delta x^2}(AK_s)^{n+1}_{i-1/2} & A_i^{n+1} + \frac{\theta\Delta t}{\Delta x^2}(AK_s)^{n+1}_{i+1/2} + \frac{\theta\Delta t}{\Delta x^2}(AK_s)^{n+1}_{i-1/2} & \frac{-\theta\Delta t}{\Delta x^2}(AK_s)^{n+1}_{i+1/2} \end{pmatrix}_{1\times3} \times \begin{pmatrix} C_{i-1}^{n+1} \\ C_i^{n+1} \\ C_{i+1}^{n+1} \end{pmatrix}_{3\times1} =$$

$$\left((AC_s)_i^n + \frac{(1-\theta)\Delta t}{\Delta x^2} \left\{ (AK_s)^n_{i+1/2}(C_s)^n_{i+1} - (AK_s)^n_{i+1/2}(C_s)^n_i - (AK_s)^n_{i-1/2}(C_s)^n_i + (AK_s)^n_{i-1/2}(C_s)^n_{i-1} \right\} \right)_{1\times1}$$

$$Neumann\ boundary\ condition: \quad \frac{C_{S\ 2}^{n+1} - C_{S\ 0}^{n+1}}{2\Delta x} = s^{n+1} \qquad O(\Delta x^2)$$

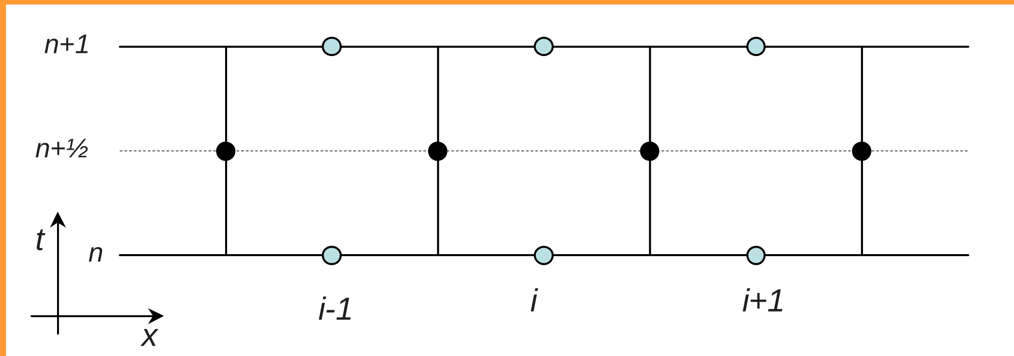
$$Dirichlet\ boundary\ condition: \quad C_s^{n+1}\ is\ known$$

Numerical Method: Advection

2nd order explicit $\frac{\partial(AC_s)}{\partial t}+\frac{\partial(QC_s)}{\partial x}=\frac{\partial}{\partial x}\left(AK_s\frac{\partial(C_s)}{\partial x}\right)+S/S=D+S/S$

$$1)\ \overline{C}_{i\pm1/2}^{n+1/2}=C_i^n+\frac{\partial C_i^n}{\partial x}\frac{\Delta x}{2}+\frac{\partial C_i^n}{\partial t}\frac{\Delta t}{2}=C_i^n+\frac{1}{2}\left(\pm1-\frac{\Delta t}{\Delta x}\frac{Q}{A}\right)\Delta C_i^n$$

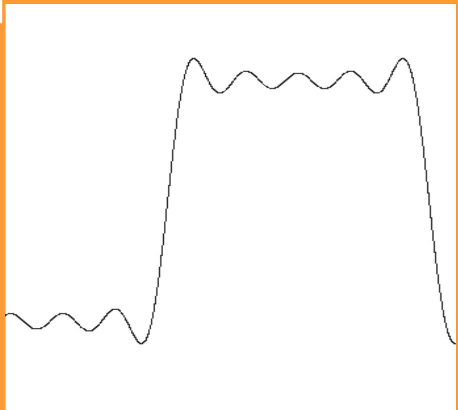
$$\begin{cases}\Delta C_i^n=\frac{\partial C}{\partial x}\Delta x=D(C_{limited})_i^n\ \Delta x\\ D(C_{limited})_i^n=Limited\quad flux\end{cases}$$



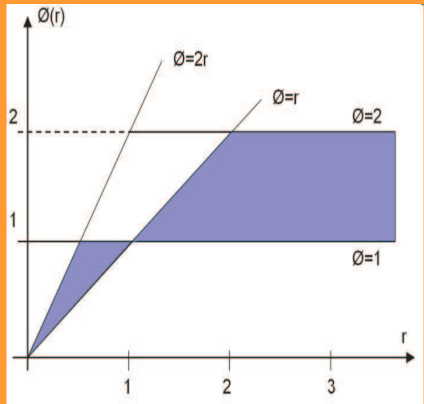
$$2)\ C_{i\pm1/2}^{n+1/2}=\overline{C}_{i\pm1/2}^{n+1/2}+\frac{\Delta t}{2A}(S_i^n+D_i^n)$$

$$3)\ AC_i^{n+1}=AC_i^n-\Delta t\times\frac{QC_{i+1/2}^{n+1/2}-QC_{i-1/2}^{n+1/2}}{\Delta x}+\frac{\Delta t}{2}\times\left[S(C_i^n)+S(\overline{C}_i^{n+1})\right]$$

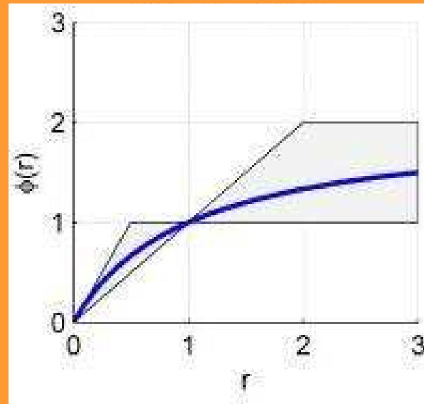
Numerical Method: Flux limiter



Gibbs phenomenon



Admissible limiters region for 2nd order schemes (Sweby 1984)



van Leer limiter (1974)

$$F\left(u_{i+\frac{1}{2}}\right) = f^{low}_{i+\frac{1}{2}} - \phi\left(r_i\right)\left(f^{low}_{i+\frac{1}{2}} - f^{high}_{i+\frac{1}{2}}\right)$$

$$F\left(u_{i-\frac{1}{2}}\right) = f^{low}_{i-\frac{1}{2}} - \phi\left(r_{i-1}\right)\left(f^{low}_{i-\frac{1}{2}} - f^{high}_{i-\frac{1}{2}}\right)$$

$$\begin{cases} r_i = \dfrac{C_i - C_{i-1}}{C_{i+1} - C_i} \\ \phi = \left[\dfrac{r + |r|}{1 + |r|}\right] \end{cases} \qquad (van\ \ Leer\ 1974)$$

Test: Mesh Convergence

- Norms as measure of functions

$$L_\infty = \|\vec{v}\|_\infty = \max |v_i|$$

(The most restrictive norm)

$$L_1 = \frac{\sum_j \|\vec{v}\|_1}{n} = \frac{\sum_j^n \sum_i |v_i|}{n}$$

$$L_2 = \frac{\sum_j^n \left(\sum_i |v_i|^2 \right)^{1/2}}{n}$$

(The less restrictive norm)

Test: Mesh Convergence II

Second order

`Table 1: Errors and Norm of Errors in Different Mesh Sizes for Θ=0.5

Num of volumes	L1	Rate of L1 change	L2	Rate of L2 change	L∞	Rate of L∞ change	Ddt/dx²	Stability
25	2.030E-03	1.094E+00	8.407E-07	5.355E+00	3.622E-04	3.844E+00	3.858E-01	O.K
50	1.855E-03	1.025E+00	1.570E-07	5.797E+00	9.422E-05	4.090E+00	1.482E+00	O.K
100	1.809E-03	1.006E+00	2.708E-08	5.760E+00	2.304E-05	4.072E+00	6.050E+00	O.K
200	1.798E-03	1.001E+00	4.701E-09	5.845E+00	5.657E-06	4.151E+00	2.445E+01	O.K
400	1.796E-03	1.000E+00	8.044E-10	6.301E+00	1.363E-06	4.611E+00	9.827E+01	O.K
800	1.795E-03	1.993E+00	1.277E-10	6.708E+00	2.956E-07	3.385E+00	3.941E+02	O.K
1600	9.008E-04	1.002E+00	1.903E-11	1.518E+00	8.732E-08	1.000E+00	1.578E+03	O.K
3200	8.991E-04	X	1.254E-11	X	8.732E-08	X	6.317E+03	O.K

Tests: Analytical Solutions I (Diffusion)

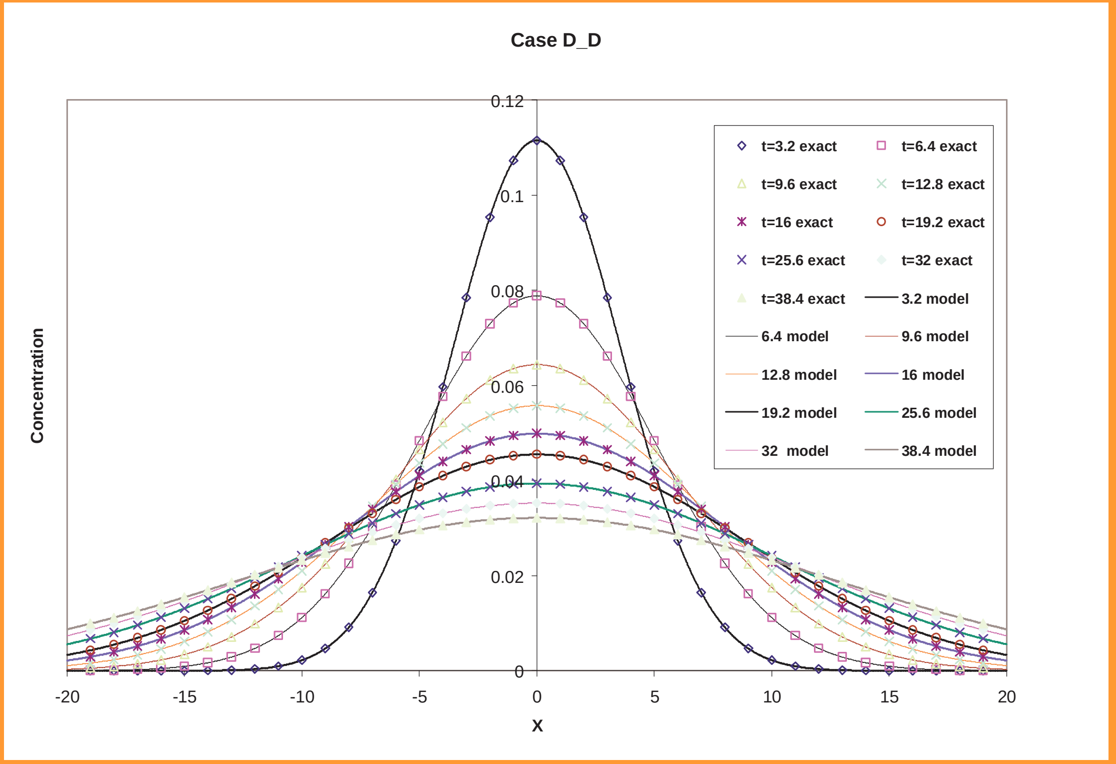
$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}$$

Subject to:

$$I.C. \quad C(x,0) = \delta(x) = \begin{cases} \infty, & x = 0 \\ 0, & x \neq 0 \end{cases}$$

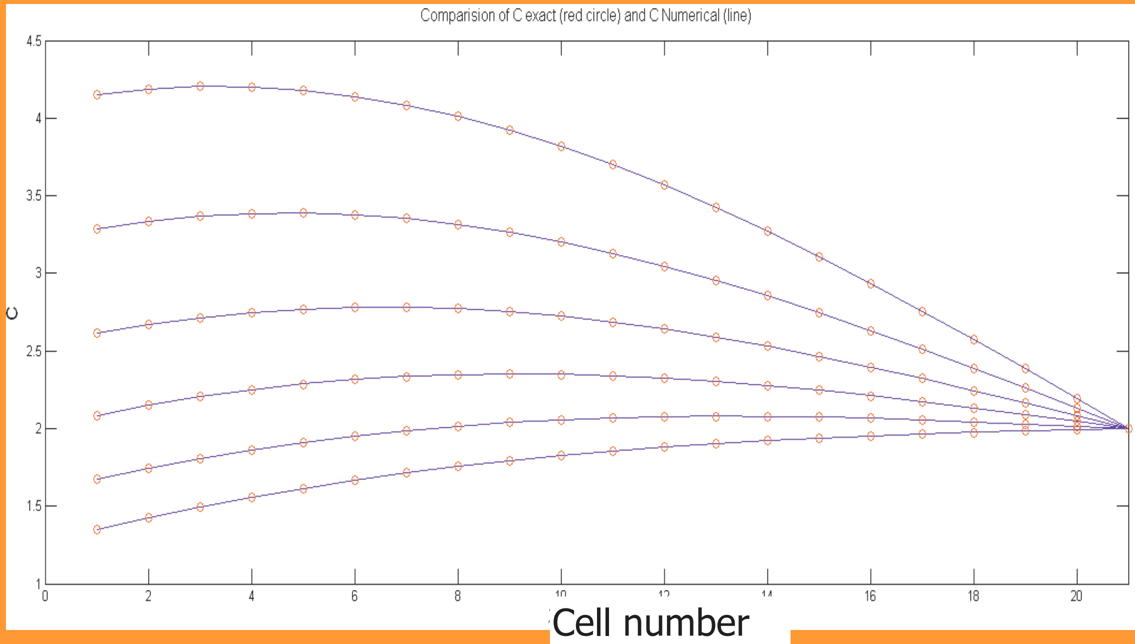
$$B.C. \quad x \to \pm\infty: \; C(x,t) = 0$$

$$C_{exact}(x,t) = \frac{e^{-\frac{x^2}{4Dt}}}{\sqrt{4\pi Dt}}$$



Tests: Analytical Solutions II (Diffusion)

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}$$



Subjected to :

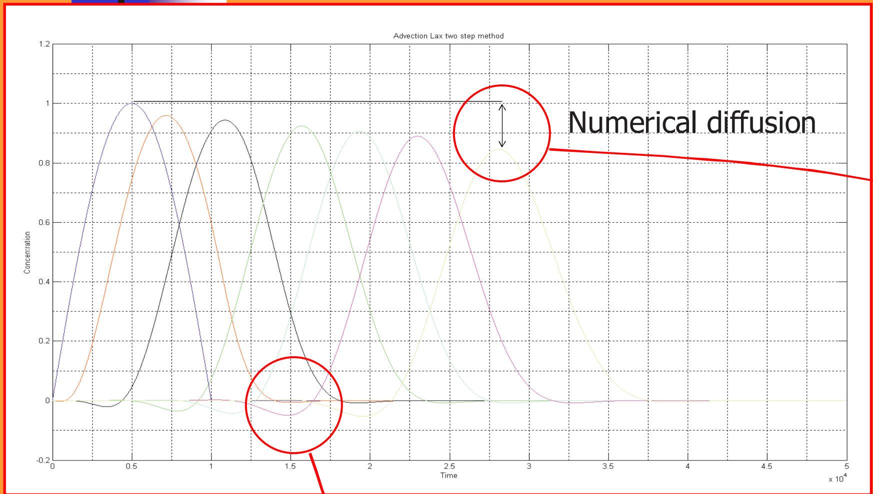
$$I.C.: c(x,0) = 2x + 4\cos\left(\frac{\pi x}{2}\right)$$

$$B.C.(x=0.1): \ \frac{\partial c}{\partial x} = 2 - 2\pi\sin(0.05\pi)\exp\left[-D\left(\frac{\pi}{2}\right)^2 t\right] \quad Neumann$$

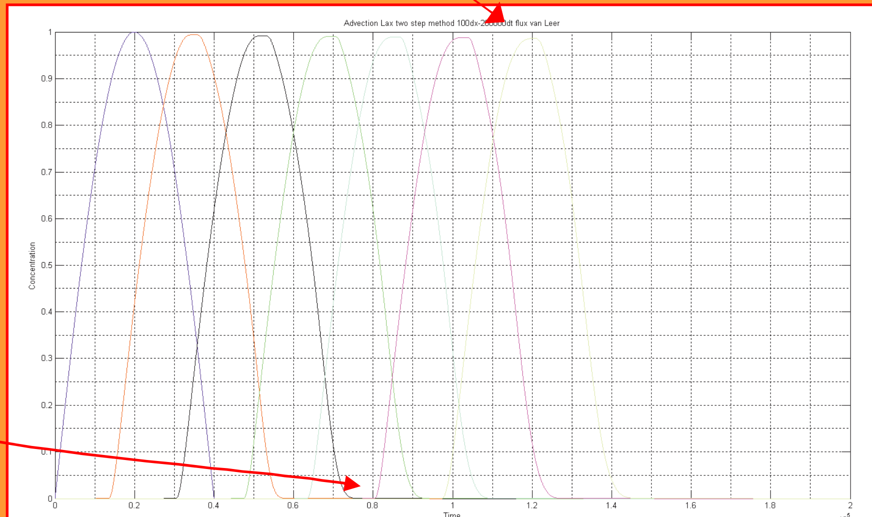
$$(x=1): \ c(1,t) = 2 \quad Dirichlet$$

$$c_{exact}(x,t) = 2x + 4\cos(0.5\pi x)\exp\left[-D\left(\frac{\pi}{2}\right)^2 t\right]$$

Tests: Advection



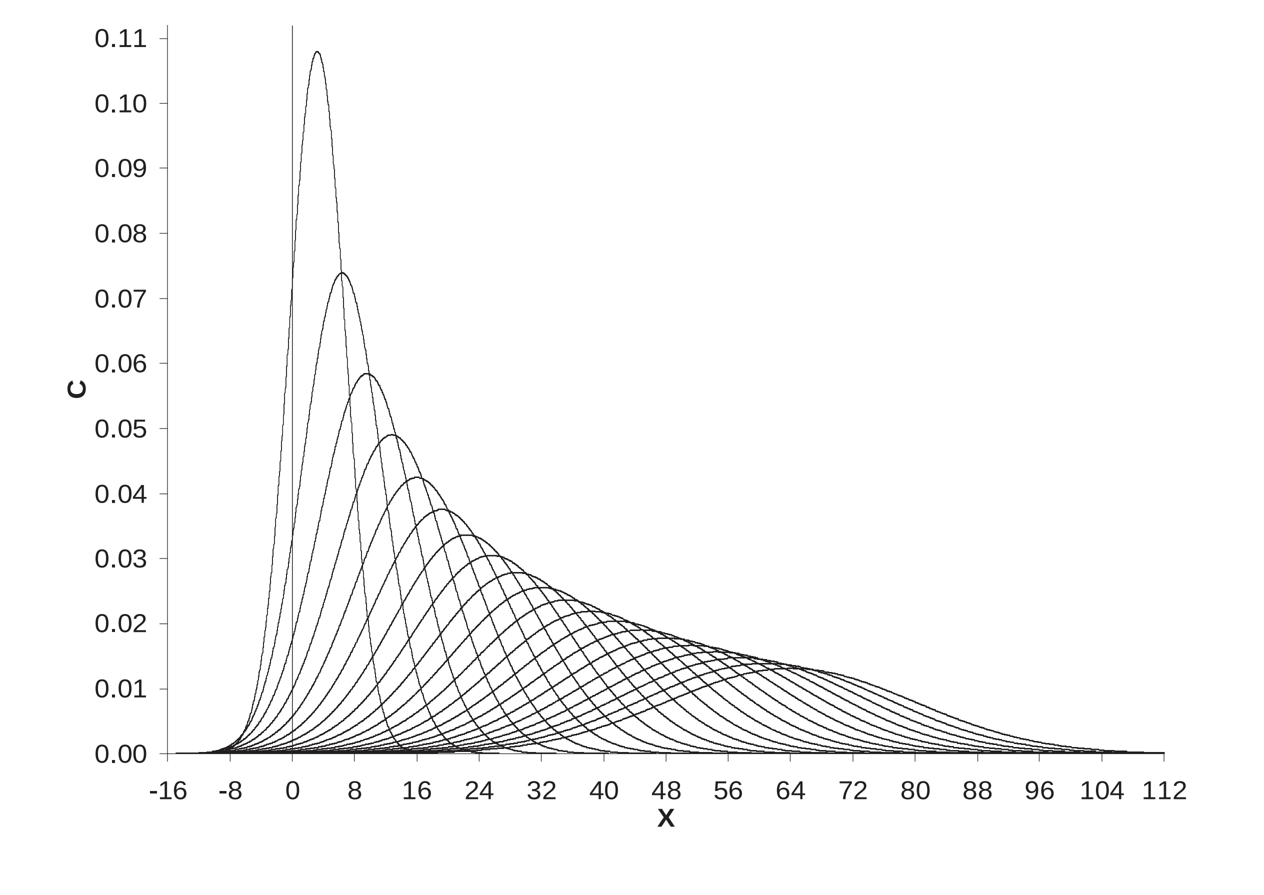
Flux limiter



Tests: A-D-R (to be included)

Source: McDonald, 2007

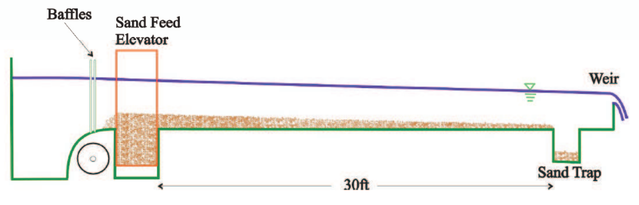
$$c = \frac{e^{-\lambda t - \frac{(x-ut)^2}{4Dt}}}{\sqrt{4\pi Dt}}$$



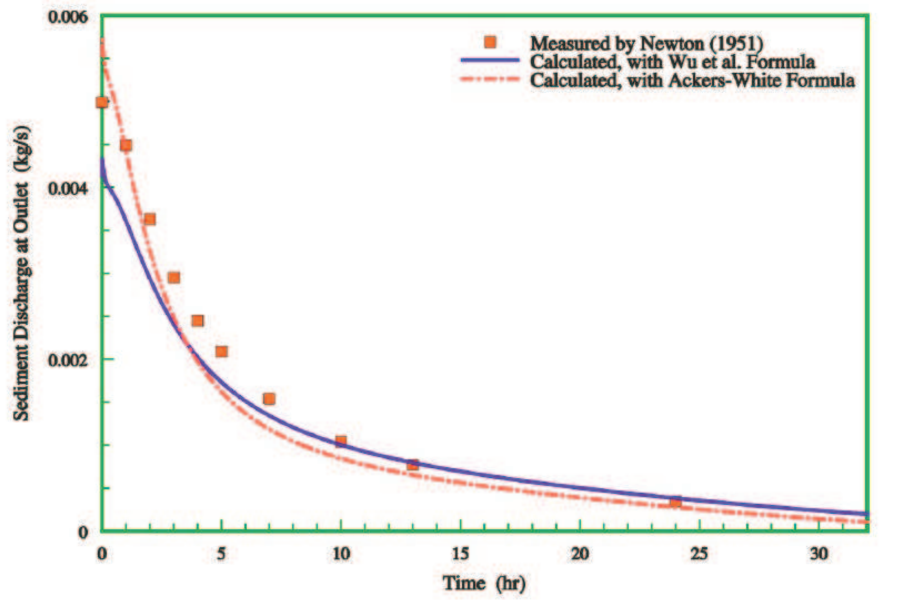
Tests: Experimental Data I (to be done)

- Comparison with experimental lab data Newton (1951).

Exp.	Flow Discharge (m^3/s)	Sediment Size (mm)	Initial Bed Slope (m/m)	Initial n_b	Final n_b	Duration (hour)
Run 1	0.00566	0.69	0.0046	0.016	0.012	24
Run 3	0.00566	0.69	0.0061	0.016	0.012	27



Configuration of Newton's (1951) Experiment



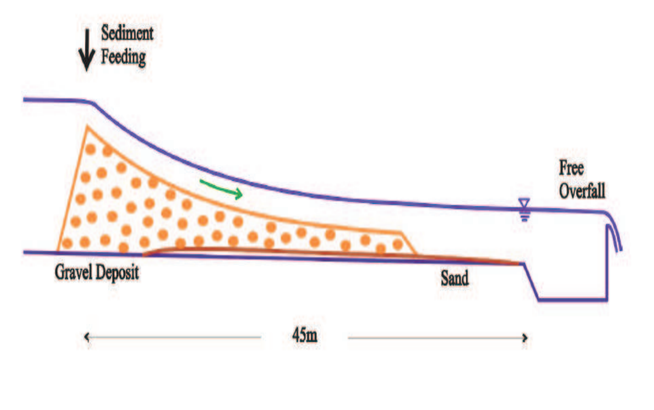
Sediment Discharges at Outlet for Newton's Exp. Run 1

Source: Wu and Vieira, 2002

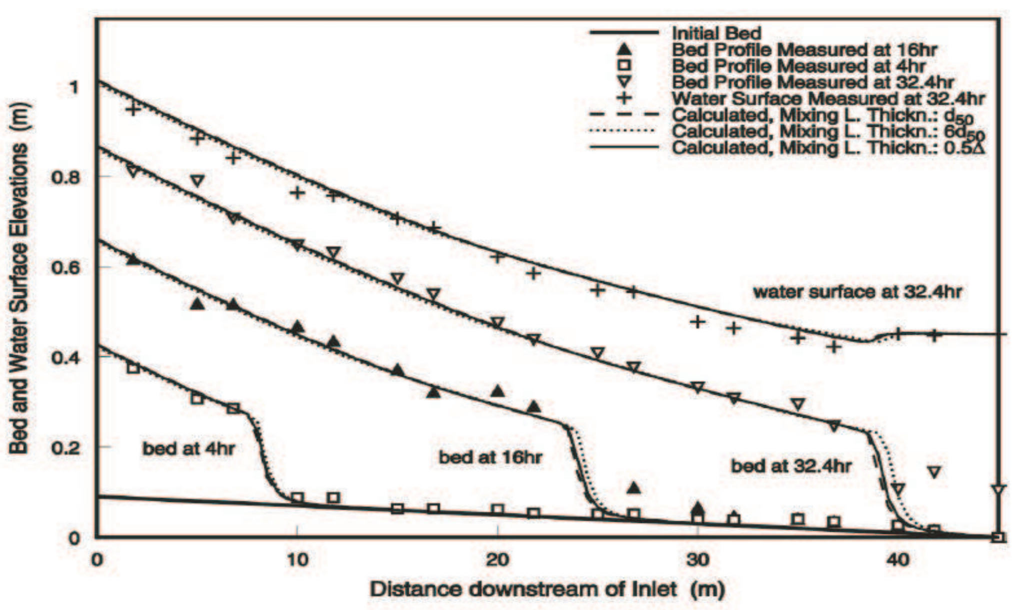
Tests: Experimental Data II (to be done)

- Comparison with experimental lab data

Cui et al. (1995/6).



Configuration of SAFHL's (1995) Experiment



Sensitivity of Bed Profile to Mixing Layer Thickness (SAFHL's Experiment Run 2)

Source: Wu and Vieira, 2002

Question: Physical Bed roughness

- $K_s = K_s'$(skin friction)+ K_s''(form drag)

1-Methods based on bed-forms and grain-related parameters such as bed-form length, height, steepness and bed-material size:

K'_s min= 0.01 m

$K'_s = 3\ d_{90}$ for $\theta < 1$ (lower regime)

$K'_s = 3\ \theta\ d_{90}$ for $\theta > 1$ (upper regime)

- Is there information on d_{90}, d_{50}, and Δ available for the Delta?

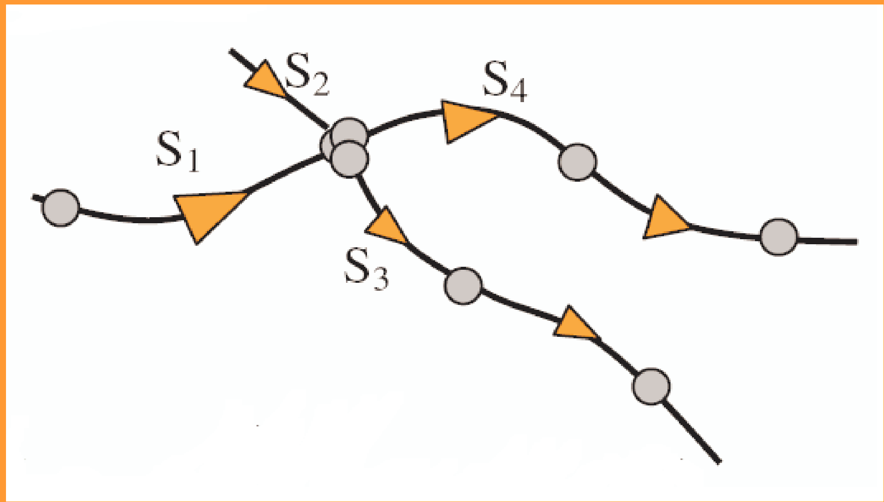
2-Methods based on integral parameters such as mean depth, mean velocity and bed material size

$$k_{s,c} = d_{50}[1 + 700(\theta - \theta_{cr})]$$

$$\theta = \tau_b / ((\rho_s - \rho) g d_{50}) \quad mobility\ \ parameter$$

Question: Physical – Distribution in Junctions

- Do you know of any studies of sediment transport at junctions in the Delta or in other systems?



Source: Mike11, DHI

$$S_3 = \frac{K_3 Q_3^{\,n3}}{K_3 Q_3^{\,n3} + K_4 Q_4^{\,n4}} (S_1 + S_2)$$

Question: Physical Treatment of Bed-load

1- Solve an advection-diffusion-reaction equation for suspended load, and use empirical formula for computing bed-load

2- Solve an advection-diffusion-reaction equation for both the bed-load and the suspended load, following the proposal by Greimann et al. (2008):

$$\frac{\partial hC}{\partial t}+\frac{\partial \cos(\alpha)\beta V_t hC}{\partial x}+\frac{\partial \sin(\alpha)\beta V_t hC}{\partial y}=\frac{\partial}{\partial x}(hfD_x\frac{\partial C}{\partial x})+\frac{\partial}{\partial y}(hfD_y\frac{\partial C}{\partial y})+S_e$$

S_e= Erosion source term
f = Transport load parameter, fraction of suspended load to total load
h = Flow depth
a = Angle of sediment transport
b = Ratio of sediment velocity to flow velocity
V_t = Total flow velocity

Question:
Physical - Entrainment & Deposition

1-In the Delta, is there any tested method for representation of settling velocity (or deposition instead) for cohesive sediment particles beyond Krone's (1962) work?

2-In the Delta, is there any tested expression for entrainment of cohesive sediment particles beyond the work by Krone (1962)?

$$E = \alpha \left[\frac{\tau_b - \tau_{cr}}{\tau_{cr}} \right]^{\beta}$$

3-To reduce the number of variables for description of cohesive sediment, which are the most important variables for the Delta?

$W_s = W_s$ (Salinity, Concentration, ...)

4-In the Delta, is there any tested method for representation of settling velocity (or deposition instead) for cohesive sediment particles?

i) W_s= constant, *ii*) $W_{s,m} = W_s(1- ac)^b$, *iii*) $W_{s,m} = Kc^m$

Question: Numerical

1-Do you know of any reliable second order methods for updating boundary conditions for advection-diffusion-reaction equations with operator splitting?

2-What order, in the splitting procedure, should we solve the advection-diffusion-reaction equations? Our initial thought is to solve advection first, then reaction and finally diffusion. Advection is always the dominant term and it should come first.

Questions:
User need/request

- What kinds of analytical tests and comparisons to data (field and laboratory) would you like to see in the STM code?
- What units for sediment/constituent concentration you would like to see in DSM2-STM? Volume per volume or mass per volume?
- Is it desirable for STM to have a feature that allows the user to select the numerical scheme to be used to solve the advection part?
- Initial non-cohesive implementation has:

1) Garcia and Parker (1991); 2) van Rijn (1984); 3) Smith and McLean (1977); 4) Zyserman and Fredsoe (1994)

Is there any other formulation you prefer to have in DSM2-STM?

Thank you!

Analog Delta Model